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OPTIMAL STRUCTURAL DESIGN OF PRESTRESSED CONCRETE BEAMS IN VIEW OF ECC 2001

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Abstract

The economy of design is a crucial factor in all engineering disciplines. In prestressed concrete structures, it is a challenge and a matter of competition among the engineers to lessen the weight of the structures and reach an optimum design. In the present work, a design procedure incorporating the generalized reduced gradient (GRG) method is used for optimization of prestressed concrete simple beams. The objective function is chosen to be the total materials' cost of the structure, subject to strength and serviceability requirements as per the latest Egyptian Code Specifications. The basic design variables categorized into continuous and discrete variables. The continuous variables are the cross sectional-dimensions and the amount of prestressing steel. The discrete variables are steel and concrete strengths. The design constraints are strength in bending, strength in shear, ductility, minimum amounts of longitudinal and shear reinforcements, and deflection. Three examples of fully prestressed simple beams are presented to demonstrate the efficiency of the optimization technique. Different starting points are tested for each example. The obtained results show the accuracy and robustness of the technique.

Keywords: Optimization; Minimum Weight; Prestressed; Concrete Beams; Design.

الملخص العربي

يقدم البحث الحلول المثلى لتصميم الكمرات الخرسانية سابقة الأجهاد طبقا لجميع اشـــتراطات الكــود المصري لتصميم وتنفيذ المنشآت الخرسانية لسنة 2001. وتمثل دالة الهدف في هذا البحـــث تكلفـــة الكمرة الخرسانية المسلحة بحديد سابق الإجهاد وتمثل متغيرات التصميم ابعاد قطاع الكمرة ومســـاحة

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الحديد سابق الإجهاد كما تمثل اشتراطات الكود قيود التصميم، وقد تم استخدام الطريقة العامة للانحدار المصغر لأيجاد التصميم الأمثل، وقد تم تطبيق الطريقة البحثية المقدمة في هذا العمل على ثلاثة أمثلة مختلفة وذلك لبيان مدى كفاءة الطريقة المقدمة وقدرتها على تحقيق تصميمات أكثر اقتصادية للكمرات الخرسانية سابقة الأجهاد.

Introduction

The topic of optimum structural design has special importance on account of the motivation of all designers to attain the optimal product in terms of cost, weight, aesthetics, reliability or a combination of these factors. Optimization of prestressed concrete structures attracts great attention as they, usually consist of repeated units and have considerable economic advantage.

A short review of previous researches on optimization of prestressed concrete structures is given in this section. Most of the work is based on ACI building code requirements. Sharaf El-Din, 2003, formulated a symmetrical prestressed concrete three-span beam as a mathematical programming problem and optimized using the Sequential Unconstrained Minimization Technique (SUMT). The beam was prismatic and had I-sections with predetermined concrete dimensions and known prestressing tendon configuration. The design variables considered in the optimization procedure included the prestressing force, tendon eccentricities, redistribution factors, and area of nonprestressing steel. The constraints were the flexural strength, ductility and serviceability. The formulation was based on ACI 318-89 requirements. The objective function was the total cost of the beam.

Fereig, 1994 and 1996, used a linear programming optimization method to establish an optimal economic design chart and to obtain the mimimum prestressing force for bridges with prestressed I-girders. The objective function was to obtain the minimum required prestressing force, while the constraints were the design conditions that were based on the requirements of the AASHTO 1992 specifications for the design of highway bridges. The solution satisfied the allowable working stress, ultimate strength, and limitations on reinforcement requirements. Shear strength, deflection, and other conditions did not control the girder selection and were checked independently.

Cohn and Lounis, 1993, presented a broad formulation of the optimal design problem for continuous prestressed concrete beams that included structural collapse modes as possible ultimate-limit-state constraints along with other standard ultimate limit state and serviceability limit state requirements. The optimization approach enabled optimum designs based on ACI 318-89 building code as well as optimum limit designs, to be done and to determine the feasibility of full redistribution designs for prestressed concrete structures.

Salama, 1991, solved the optimization of simply supported prefabricated prestressed concrete beams using nonlinear programming techniques. The method of Fletcher and Powell was used in his work with an interior penalty function of Fiacco-Mccormik. The method of analysis used was mainly based on ACI 318-89 provisions. Beams with different spans were designed and clearly indicated that the benefit gained by the optimized weight reduction ranged from 5%-52%.

Hassanain and Reda, 2002, introduced a rigorous and systematic procedure using mathematical optimization techniques for the design of high-performance concrete and fibre-reinforced polymers. The procedure was used to develop an optimization system that can be utilized to carry out cost effectiveness studies, and to develop preliminary design charts and guidelines according to the Canadian Highway Bridge Design Code Provisions.

In this paper, a design procedure that is based on the latest Egyptian Code specifications ECC'01 (ECC'01 2001) is presented. A design procedure incorporating the GRG method is used for nonlinear optimization of prestressed concrete simple beams (Abadie and Carpentier 1969) and (Lasdon and Warren 1978). The objective function considered is the total materials cost of the beam. This function is minimized subject to strength and serviceability requirements. The basic design variables considered in the optimization procedure are divided into continuous and discrete variables. The continuous variables are the cross-sectional dimensions and the amount of prestressing and nonprestressing steels. The discrete variables are steel and concrete strengths. The design constraints are strengths in bending and shear, ductility, minimum amounts of longitudinal and shear reinforcements, and deflection.

A computer program incorporating the design variables and constraints has been developed. The program assigns comprehensive sections that can be easily changed to rectangular, T-, inverted T- and I-sections. The properties of the mentioned sections are internally calculated. In the case of using other sections not assigned in the program, the user should assign all the required dimensions and section characteristics.

Three examples of simply supported beams having different shapes of cross-sections and spans are studied to verify the optimization technique. Three different starting points are applied for each example to clarify the efficiency of the optimization method. All results show improvements in the objective functions for all cases, with different starting points.

Analysis Problem

Most structures are designed on trial-and-error basis. A preliminary design is estimated and analyzed. If it is satisfactory, it is considered a feasible design. If the trial design is not satisfactory, the designer has to change it and repeat the analysis until a feasible one is obtained. Usually, there are an infinite number of feasible designs, and the designers strive to find the optimal within the time they have available.

A simple beam with span 'L' subject to uniformly distributed load is shown in Fig.(1). A comprehensive cross-section with the design variables is shown in Fig. (2).

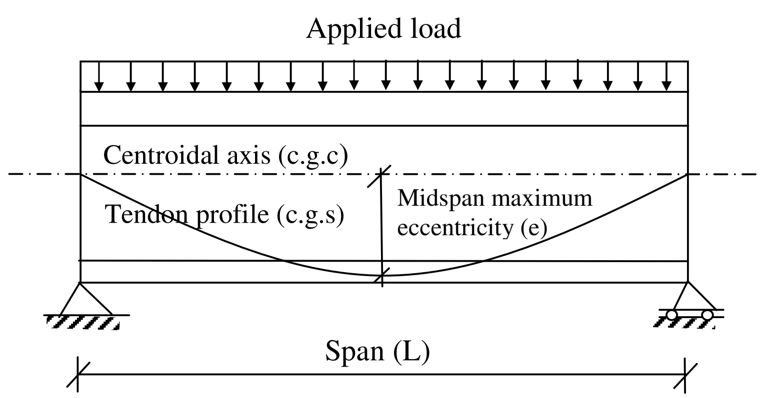


Fig. (1) Parabolic Tendon Profile for Simply Supported Beam

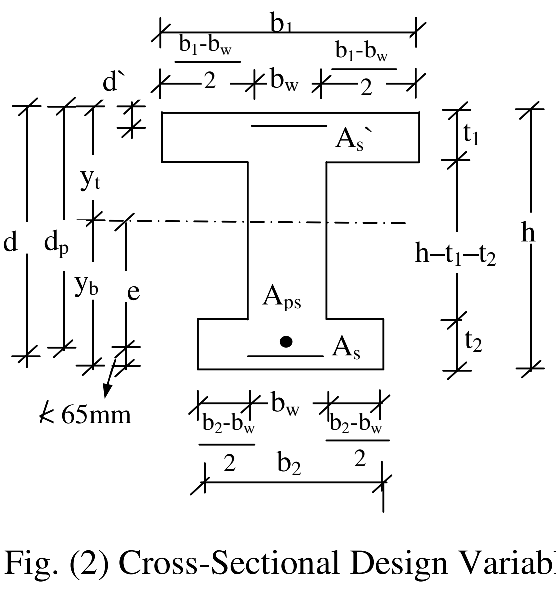
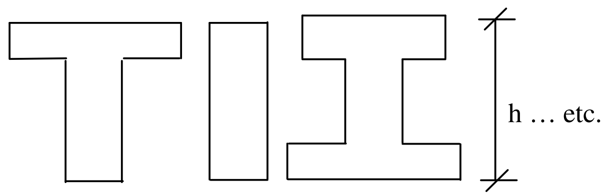


Fig. (2) Cross-Sectional Design Variables

For such beams, it is economical to use the largest possible eccentricity that can be accommodated within the cross-section, since it minimizes the required prestressing force. Therefore, the midspan eccentricity is given as a preassigned design value in this work. A parabolic tendon profile with zero eccentricity at the supports and the maximum possible eccentricity at midspan is shown in Fig. (1). Figure (3) summarizes the design process of simply supported prestressed concrete beams (Nilson 1978), (Lin and Burns 1981), and (Collins and Mitchell 1991).

Design Problem

Choice of the Cross-Section and Material Properties



- Preliminary choice of the overall member depth based on the typical span-to-depth ratios.
- Determine the other cross-sectional dimensions.
- Choose the tendon profile.
- Calculate the member dead load, w_D.

Choice of Prestressing

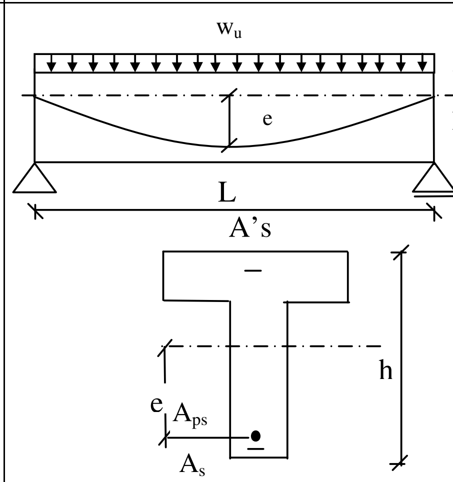
Sensitive to Cracking	Sensitive to Deflection	Controlled by Strength
Determine the values of prestressing force and eccentricity that will satisfy the stress limits at midspan of a simply supported beam at both the initial and final stages. For example $P_e \geq \dfrac{M_{total} - \dfrac{I_c}{y_b} F_{2s}}{e + \dfrac{I_c}{A_c y_b}}$	To have zero deflection under uniform dead load, w_D, for beams prestressed with a parabolic profile $P_e.e = \dfrac{w_D L^2}{8}$	 $A_{ps}\left(\dfrac{f_{ps}}{\gamma_{ps}}\right)\left(d_p - \dfrac{a}{2}\right) + A_s\left(\dfrac{f_y}{\gamma_s}\right)\left(d - \dfrac{a}{2}\right) = \dfrac{w_u L^2}{8}$

Fig. (3) Design Procedures for Prestressed Concrete Beams

Formulation of the Optimal Design Problem

The formulation of an optimal design problem requires identification of a set of design variables that describe the structure, design constraints that must be satisfied, and an objective function that measures the merits of alternate designs. The design variables, parameters and constraints are expressed in a suitable format for the application of GRG technique that is used in this study to find the optimum design of the prestressed concrete beams.

Preassigned Parameters and Design Variables

The preassigned parameters of the problem are the span length 'L', and the distance at midspan from the centroid of the beam to the centroid of the prestrening steel 'e'. The centroid of the steel is at the centroid of the section at the span end. Additional design preassigned parameters are the characteristic concrete strength at transfer and at 28 days, the strength of prestressing steel and of nonprestressing steel reinforcement, the modulus of elasticity of the concrete and the concrete creep factor. The design variables are b_1, b_2, b_w, t_1, t_2, h which define the cross-section dimensions and A_{ps}, A_s, $A_{s'}$ which define the areas of prestressing and nonprestressing steel (see Fig.2).

Design Constraints

The optimum design of the prestressed concrete beams is the one with minimum cross-sectional area and material costs which satisfies all design constraints. Theses constraints in case of simple beams as per ECC'01 can be linearized and categorized in the following form: -

1- Permissible Stresses in Concrete

The ECC'01 specifies two stages namely (in case of simple beam); (see Fig. 4)

(a) Stresses at transfer (initial stage)

at top $$P_i\left(-\frac{1}{A_c}+\frac{e.y_t}{I_c}\right)-\frac{M_\circ.y_t}{I_c}\leq 0.22\sqrt{f_{cui}} \tag{1}$$

at bottom $$P_i\left(-\frac{1}{A_c}-\frac{e.y_b}{I_c}\right)+\frac{M_\circ\cdot y_b}{I_c}\geq -0.45\sqrt{f_{cui}} \tag{2}$$

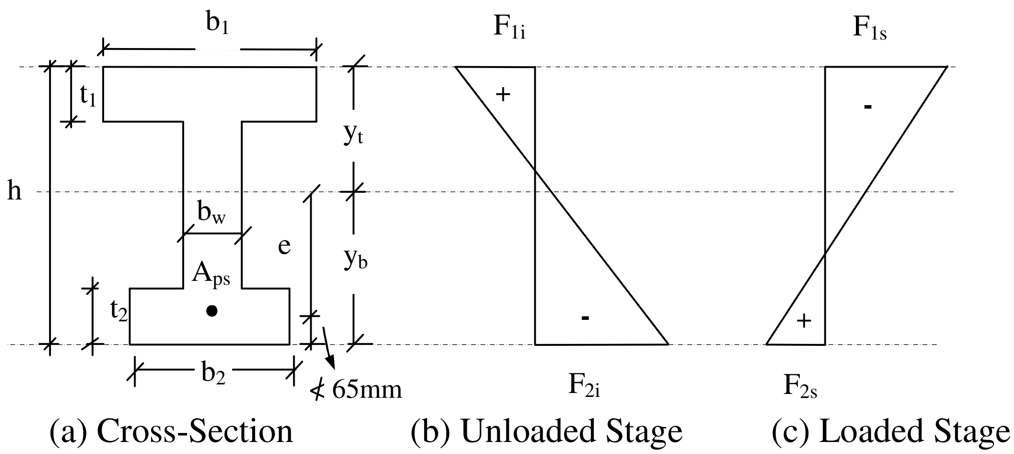
(b) Stresses at service stage (final stage)

at top $$RP_i\left(-\frac{1}{A_c}+\frac{e.y_t}{I_c}\right)-\frac{\overline{M}_{total}\cdot y_t}{I_c}\geq -0.35\ f_{cu} \tag{3}$$

or $$RP_i\left(-\frac{1}{A_c}+\frac{e.y_t}{I_c}\right)-\frac{M_{total}.y_t}{I_c}\geq -0.40\ f_{cu} \quad (4)$$

at bottom $$RP_i\left(-\frac{1}{A_c}-\frac{e.y_b}{I_c}\right)+\frac{M_{total}.y_b}{I_c}\leq 0.44\sqrt{f_{cu}} \quad (5)$$

It should be mentioned here that, the elastic stresses at the span end will not be critical since the centroid of the prestressing steel is at the centroid of concrete section.



F_1 = The stress at top fiber (N/mm^2), F_2 = The stress at bottom fiber (N/mm^2).
i = Subscript denoted initial stage, s = Subscript denoted service stage.

Fig.(4) Elastic Stresses in an Uncracked Prestressed Beam

2. Permissible Stresses in Prestressed Tendons

The area of prestressing tendon in tension zone is

$$A_{ps}\geq\frac{P_i}{0.70f_{pu}} \quad (6)$$

where P_i is the minimum value obtained from Eqs. (1 to 5)

$$if\ \frac{RP_i}{A_{ps}f_{pu}}\geq 0.50$$

$$then\ \ f_{ps}=f_{pu}\left(1-\eta_p\left[\mu_p\frac{f_{pu}}{f_{cu}}+\frac{d}{d_p}(w-w')\right]\right) \quad (7)$$

$$where\left[\mu_P \frac{f_{pu}}{f_{cu}} + \frac{d}{d_p}(w - w')\right] \geq 0.17,$$

$$d' \leq 0.15 d_p \left(when\, d' > 0.15 d_p\, then\, w' = zero \right), and \tag{8}$$

$$w = \mu \frac{f_y}{f_{cu}} \; and \; w' = \mu' \frac{f_y}{f_{cu}}$$

3. Flexural Strength

$For\ a \leq t_1$

$$a = \frac{A_{ps}\left(\frac{f_{ps}}{\gamma_{ps}}\right) + A_s\left(\frac{f_y}{\gamma_s}\right) - A_s'\left(\frac{f_y}{\gamma_s}\right)}{0.67\,\frac{f_{cu}}{\gamma_c}\,b_1} \tag{9}$$

$$M_u \leq A_{ps}\left(\frac{f_{ps}}{\gamma_{ps}}\right)\left(d_p - \frac{a}{2}\right) + A_s\left(\frac{f_y}{\gamma_s}\right)\left(d - \frac{a}{2}\right) - A_{s'}\left(\frac{f_y}{\gamma_s}\right)\left(\frac{a}{2} - d'\right) \tag{10}$$

$For\ a > t_1$

$$a = \frac{A_{ps}\left(\frac{f_{ps}}{\gamma_{ps}}\right) + A_s\left(\frac{f_y}{\gamma_s}\right) - \frac{0.67 f_{cu}}{\gamma_c}(b_1 - b_w)t_1 - A_{s'}\left(\frac{f_y}{\gamma_s}\right)}{0.67\,\frac{f_{cu}}{\gamma_c}\,b_w} \tag{11}$$

$$M_u \leq A_{ps}\left(\frac{f_{ps}}{\gamma_{ps}}\right)\left(d_p - \frac{t_1}{2}\right) + A_s\left(\frac{f_y}{\gamma_s}\right)\left(d - \frac{t_1}{2}\right) - 0.67\frac{f_{cu}}{\gamma_c} b_w (a - t_1)\frac{a}{2}$$

$$- A_{s'}\left(\frac{f_y}{\gamma_s}\right)\left(\frac{t_1}{2} - d'\right) \tag{12}$$

where

$$M_u = \frac{w_u . L^2}{8} \tag{13}$$

$$w_u = 1.40\, w_D + 1.60\, w_L \tag{14}$$

4. Maximum Flexural Reinforcement (Ductility check)

For rectangular section
$$w_p + \frac{d}{d_p}\left(w - w'\right) \le 0.28 \qquad (15)$$

For flanged section
$$w_{pw} + \frac{d}{d_p}\left(w_w - w'_w\right) \le 0.28 \qquad (16)$$

where
$$w_p = \mu_p \frac{f_{ps}}{f_{cu}}, w = \mu \frac{f_y}{f_{cu}}, w' = \mu' \frac{f_y}{f_{cu}} \qquad (17)$$

5. Minimum Flexural Reinforcement

$For\ a \le t_1$

$$A_{ps}\left(\frac{f_{ps}}{\gamma_{ps}}\right)\left(d_p - \frac{a}{2}\right) + A_s\left(\frac{f_y}{\gamma_s}\right)\left(d - \frac{a}{2}\right) - A_{s'}\left(\frac{f_y}{\gamma_s}\right)\left(\frac{a}{2} - d'\right) \ge 1.20\, M_{cr} \qquad (18)$$

$For\ a > t_1$

$$A_{ps}\left(\frac{f_{ps}}{\gamma_{ps}}\right)\left(d_p - \frac{t_1}{2}\right) + A_s\left(\frac{f_y}{\gamma_s}\right)\left(d - \frac{t_1}{2}\right) - 0.67\,\frac{f_{cu}}{\gamma_c} b_w \left(a - t_1\right)\frac{a}{2}$$
$$- A_{s'}\left(\frac{f_y}{\gamma_s}\right)\left(\frac{t_1}{2} - d'\right) \ge 1.20 M_{cr} \qquad (19)$$

where

$$M_{cr} = \frac{I_c}{y_b}\left(0.60\sqrt{f_{cu}} + f_{pce} - f_{cd}\right) \qquad (20)$$

$$f_{pce} = \frac{RP_i}{A_c} + \frac{RP_i.e.y_b}{I_c} \qquad (21)$$

$$f_{cd} = \frac{M_d . y_b}{I_c} \qquad (22)$$

6. Shear Strength

The current version of the Egyptian Code requires that the value of ${f_{cu}}^{0.5}$ used in all shear design equations be limited to 7.0 N/mm^2. This limit can be waived if additional web reinforcement is provided.

$$if\ \frac{Q_{du}}{b_w d} \le q_{cu} \tag{23}$$

then use minimum shear reinforcement

where

$$Q_{du} = w_u\left(\frac{L}{2} - \frac{h}{2}\right) - RP_i\ Sin\left(\frac{4.e}{L}\right) \tag{24}$$

Calculation of q_{cu}

(a)
$$q_{cu} = 0.045\sqrt{f_{cu}} + \frac{5Q_u d_p}{M_u} \tag{25}$$

but
$$0.125\sqrt{f_{cu}} \le q_{cu} \le 0.35\sqrt{f_{cu}}\ \&\ \frac{Q_u d_p}{M_u} \le 1.0 \tag{26}$$

where
$$Q_u = w_u\left(\frac{L}{2} - \frac{h}{2}\right) \tag{27}$$

(b) or the smaller value q_{ci} and q_{cw}

Flexural Shear cracking

$$q_{ci} = 0.045\sqrt{f_{cu}} + q_d + q_i\frac{M_{cr}}{M_{max}} \ge 0.125\sqrt{f_{cu}} \tag{28}$$

$$where\quad M_{max} = \frac{w_u Lh}{4} - \frac{w_u h^2}{8} \tag{29}$$

$$q_d = \frac{w_D 0.5(L-h)}{b_w d} \tag{30}$$

$$q_i = \frac{Q_{du}}{b_w d} \tag{31}$$

and web-shear cracking

$$q_{cw} = 0.27\sqrt{f_{cu}} + 0.30 f_{pcc} + q_{pv} \tag{32}$$

where
$$f_{pcc} = \frac{RP_i}{A_c} \tag{33}$$

$$q_{pv} = \frac{RP_i \, Sin(\frac{4.e}{L})}{b_w d_p} \tag{34}$$

$$\mu''_{min} = \frac{A_{st}}{b_w . s} = \frac{0.40}{f_y} \tag{35}$$

$$\not\subset 0.15 \text{ for normal smooth reinforcement}$$
$$\not\subset 0.10 \text{ for deformed high - grade reinforcement}$$
$$\not\subset 5\phi 6 \text{ mm/m}$$

$$If \ \frac{Qd_u}{b_w d} > q_{cu} \tag{36}$$

$$then \ q_{su} = \frac{Qd_u}{b_w d} - 0.50 q_{cu} \tag{37}$$

$$\frac{Qd_u}{b_w d} \leq 0.70 \sqrt{\frac{f_{cu}}{\gamma_c}} \text{ or } 3N/mm^2 \text{ whichever is smaller} \tag{38}$$

$$\mu'' = \frac{A_{st}}{b_w . s} = \frac{q_{su}}{(f_y / \gamma_s)} \tag{39}$$

7. Deflection

The Egyptian Code requires that the deflections of prestressed concrete beams due to both short-term live loads and long-term dead loads and sustained live loads should be calculated. In calculating long-term camber and deflections, creep and shrinkage of the concrete and relaxation of the steel are to be taken into account. The computed deflections for simply supported prestressed concrete beams having parabolic tendon profile and subjected to uniformly distributed loads must not exceed the limits given by the following equations.

$$-\Delta_{pe} - \left(\frac{\Delta_{pi} + \Delta_{pe}}{2} \right) k_{cr} + \left(\Delta_{wo} + \Delta_d + \Delta'_L \right)\left(1 + k_{cr} \right) \leq \frac{L}{350} \tag{40}$$

$$-\Delta_{pe} - \left(\frac{\Delta_{pi} + \Delta_{pe}}{2} \right) k_{cr} + \left(\Delta_{wo} + \Delta_d + \Delta'_L \right)\left(1 + k_{cr} \right) + \Delta''_L \leq \frac{L}{250} \tag{41}$$

$$where \ \Delta_{pi} = \frac{5 p_i e L^2}{48 E_{ci} I_c}, \tag{42}$$

$$\Delta_{wo} = \frac{5w_o\ L^4}{384E_{ci}I_c}, \tag{43}$$

$$\Delta_d = \frac{5w_d\ L^4}{384E_cI_c}, \tag{44}$$

$$\Delta'_L = \frac{5w'_L\ L^4}{384\ E_{\ c}I_c}, \tag{45}$$

$$\Delta''_L = \frac{5w''_L\ L^4}{384\ E_{\ c}I_c}, \tag{46}$$

$$and\ \Delta_{pe} = \Delta_{pi} \times R \tag{47}$$

Objective Function

The objective function 'f' is chosen to be the ratio of the total cost of the girder to the cost of concrete volume. A comprehensive survey for both national and international marketing prices shows that the ratio of the cost of prestressing tendons in tons to the cost of cubic meter of concrete is about 30. Therefore, the objective cost function is defined as follows:

$$f\ = 1 + 234\frac{A_{ps}L'}{A_cL} \tag{48}$$

The above cost function is minimized under all relevant constraints given by Eqs. (1 to 47). It should be mentioned that, the total number of design variables is nine. However, the number of design constraints is twenty-three.

The Generalized Reduced Gradient (GRG) Method

The generalized reduced gradient (GRG) method is an extension of the reduced gradient method to accommodate nonlinear inequality constraints. In this method, a search direction is found, such that for any small move, the current active constraints remain precisely active. If some active constraints are not precisely satisfied due to nonlinearity of constraint functions, the Newton-Raphson technique is used to return to the constraint boundary, Arora, Jasbir. S., (1989).

Abadie, J. and Carpentier, J., (1969) partition the design variable vector **x**, of n variables, as $[\mathbf{y}^T , \mathbf{z}^T]^T$ where $\mathbf{y}_{(n-p)}$ and $\mathbf{z}_{(p)}$ are vectors of independent and dependent design variables, respectively. First order changes in the objective and constraint functions (treated as equalities) are gives as

$$\Delta f = \frac{\partial f^T}{\partial \mathbf{y}} \Delta \mathbf{y} + \frac{\partial f^T}{\partial \mathbf{z}} \Delta \mathbf{z} \tag{49}$$

$$\Delta h_i = \frac{\partial h_i^T}{\partial \mathbf{y}} \Delta \mathbf{y} + \frac{\partial h_i^T}{\partial \mathbf{z}} \Delta \mathbf{z} \tag{50}$$

Any change in the variables must keep the current equalities satisfied at least to first order, i.e. $\Delta h_i = 0$. Therefore, Eq. (50) can be written in the matrix form as

$$\mathbf{A}^T \Delta \mathbf{y} + \mathbf{B}^T \Delta \mathbf{z} = \mathbf{0}, \quad \text{or} \quad \Delta \mathbf{z} = -(\mathbf{B}^{-T} \mathbf{A}^T)\, \Delta \mathbf{y} \tag{51}$$

Where columns of matrices $\mathbf{A}_{((n-p)xp)}$ and $\mathbf{B}_{(pxp)}$ contain gradients of equality constraints with respect to **y** and **z**. Eq. (51) can be viewed as the one that determines $\Delta\mathbf{z}$ (change in the dependent variable vector) when $\Delta\mathbf{y}$ (change in the independent variable vector) is satisfied. Substituting $\Delta\mathbf{z}$ form Eq. (51) into Eq. (49), Then

$$\Delta f = \left(\frac{\partial f^T}{\partial \mathbf{y}} - \frac{\partial f^T}{\partial \mathbf{z}} \mathbf{B}^{-T} \mathbf{A}^T \right) \Delta \mathbf{y}, \quad \text{or} \quad \frac{df}{d\mathbf{y}} = \frac{\partial f}{\partial \mathbf{y}} - \mathbf{A}\mathbf{B}^{-1} \frac{\partial f}{\partial \mathbf{z}} \tag{52}$$

For a trial step size α, the design variables are updated using $\Delta\mathbf{y} = -\alpha\, df/d\mathbf{y}$ and $\Delta\mathbf{z}$ from Eq. (51). If the trial design is not feasible, then independent design variables are considered to be fixed and dependent variables are changed iteratively by applying the Newton-Raphson method until we get a feasible design point. If the new feasible design satisfies the descent condition, then line search is terminated; otherwise, the previous trial step size is discarded and the procedure is repeated with a reduced step size.

Applications

The generalized reduced gradient (GRG) nonlinear optimization method is utilized for design of several prestressed concrete beams. The optimization algorithm seeks a feasible solution, if one is not provided and then retains feasibility as the objective function is improved. A robust quasi-Newton algorithm is implemented for determining a search direction. Three simply supported beams subject to uniformly distributed load are investigated. These examples covered R-, T-, and I-sections. The beams have spans ranging from 12000 mm to 35000 mm. Three different starting points are applied for each example to demonstrate the robustness of the algorithm. The following subsections summarize the studied examples.

Example 1

A simple beam with span length 12000 mm and having a rectangular cross section is selected. The loads and material properties are shown in Fig. (5). Three different starting points have been applied. All starting points are convergencing to the same dimensions and objective function as illustrated in Table (1) and Fig. (6). The developments of h and A_{ps} are shown in Fig. (7).

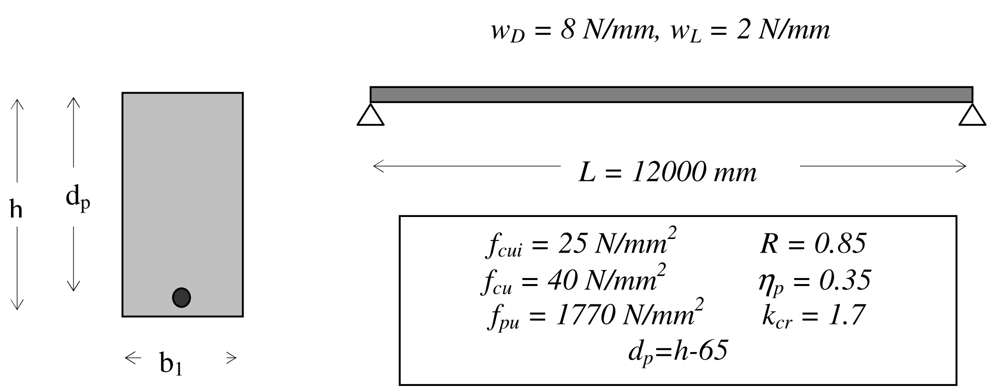


Fig. (5) Simple Beam with Rectangular Cross Section (Example 1)

Table (1) Starting and Optimum Dimensions (Example 1)

Sec.	Starting Point No.		
	I	**II**	**III**
b_1	300	300	350
h	700	900	1100
Aps	518.2	356.02	236.74
Optimum Values			
b_1	250	250	250
h	648.43	648.43	648.43
Aps	492.45	492.45	492.45

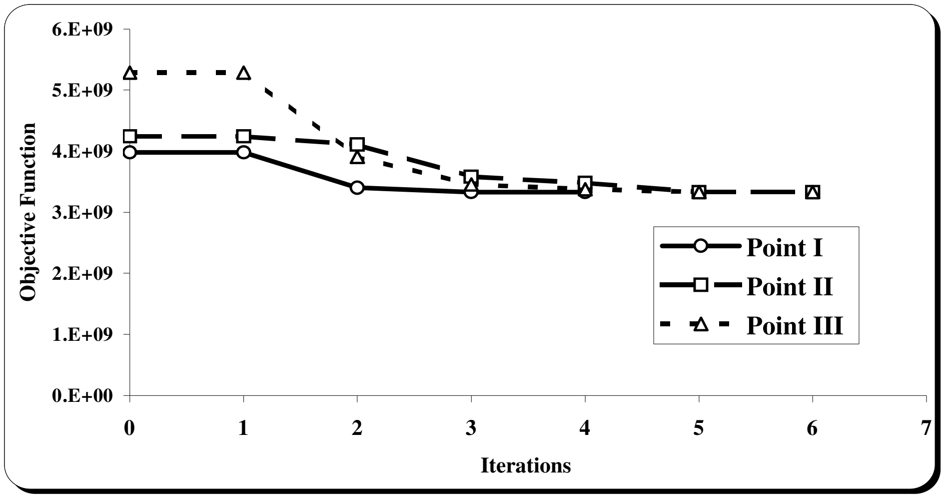


Fig. (6) Development of the Objective Function (Example 1)

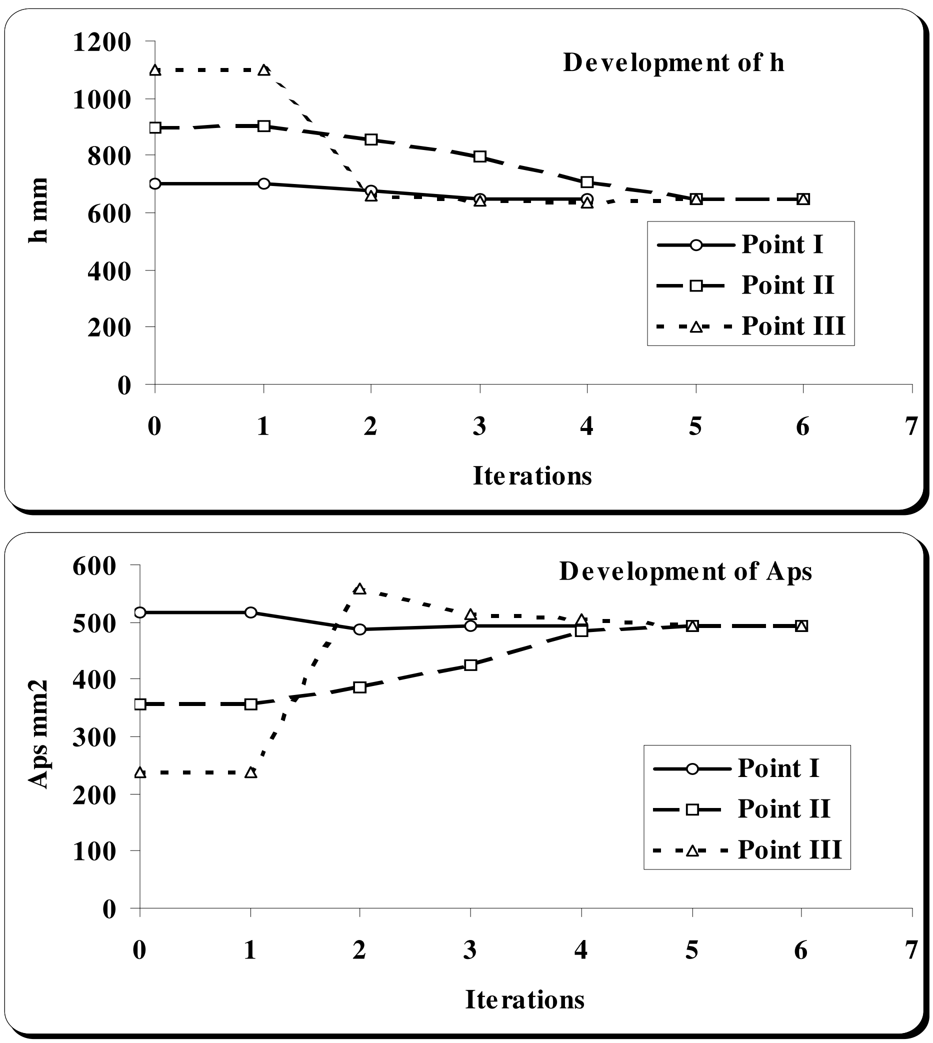


Fig. (7) Development of the Design Variables (Example 1)

Example 2

A simply supported beam having T-section and span of 20000 mm is shown in Fig. (8). The design variables for the different starting points are given in Table (2). The development and convergence of the objective function and design variables are illustrated in Figs. (9 and 10) and Table (2). It clearly shown that all starting points are minimized to the same value.

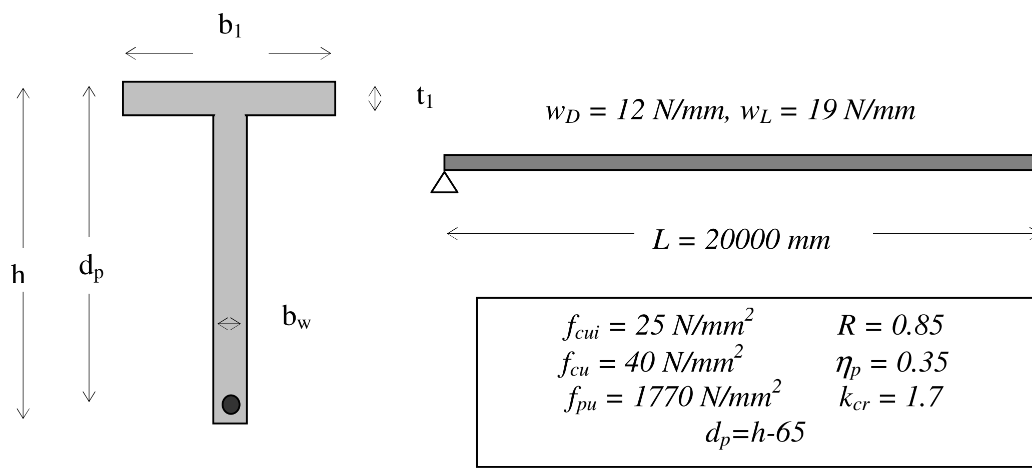


Fig. (8) Simple Beam with T- Cross Section (Example 2)

Table (2) Starting and Optimum Dimensions (Example 2)

Sec.	Starting Point No.		
	I	**II**	**III**
b_w	300	350	400
h	1800	2500	3500
b_1	1500	2000	2300
t_1	150	250	300
Aps	1393.7	913.94	336.67
Optimum Values			
b_w	250	250	250
h	2009.82	2009.87	2009.35
b_1	488.06	488.21	488.54
t_1	150	150	150
Aps	1192.75	1192.81	1192.94

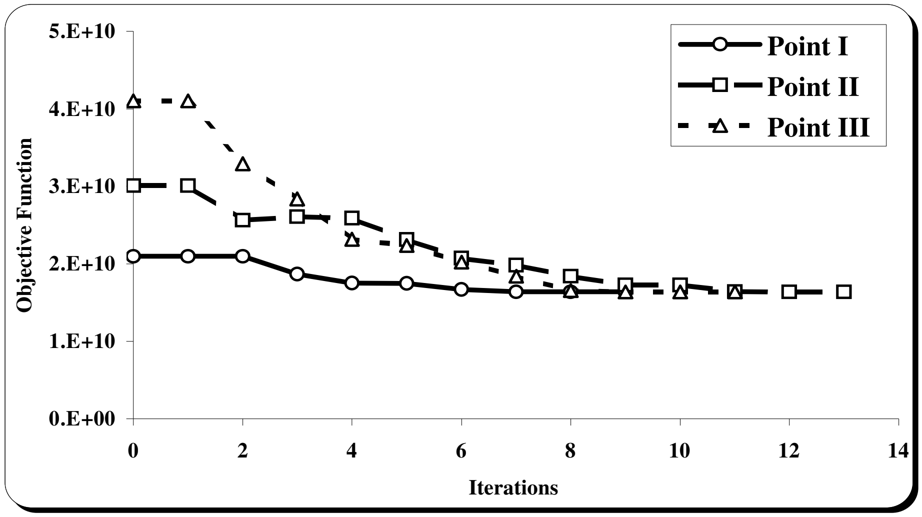
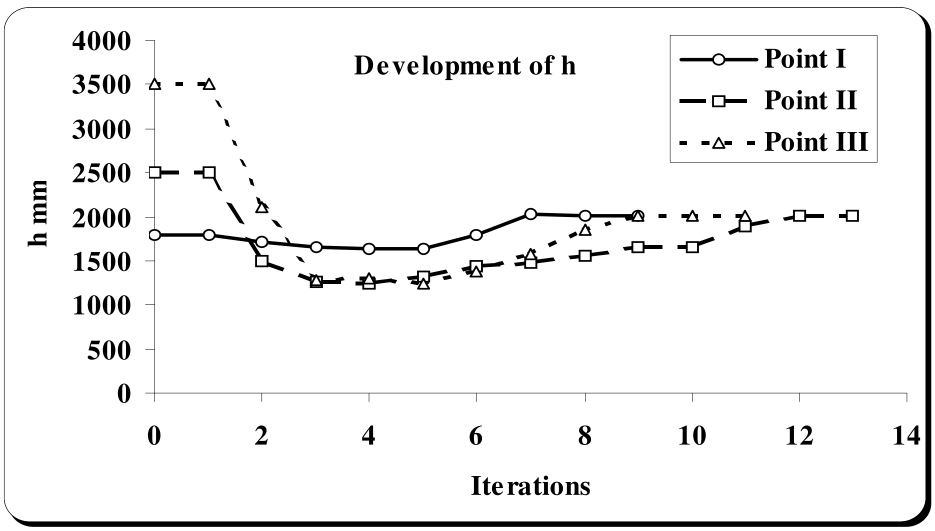


Fig. (9) Development of the Objective Function (Example 2)



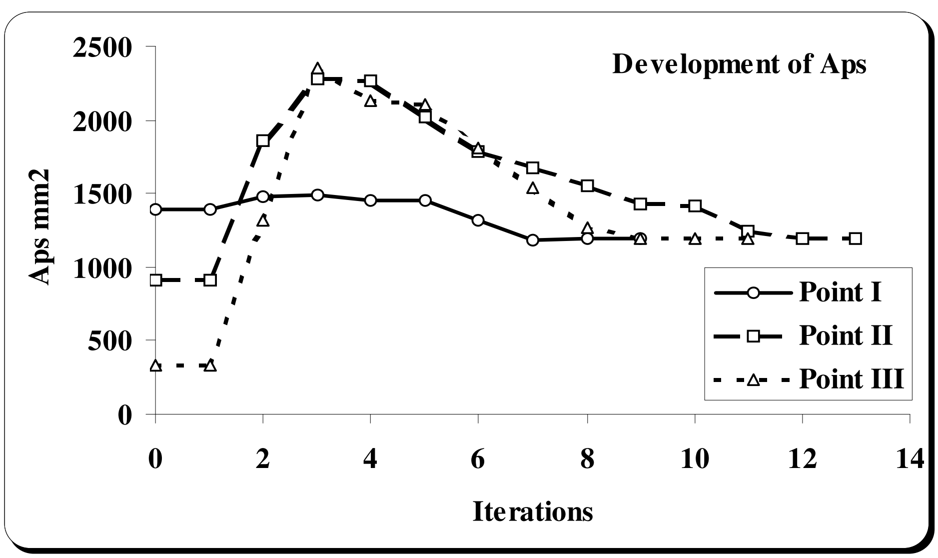


Fig. (10) Development of the Design Variables (Example 2)

Example 3

Figure (11) shows a simple beam with a span of 35000 mm and has I-section. The concrete and steel strengths are shown in this figure. Figures (12 & 13) and Table (3) show the history of design variables and the objective function. From the observation of these figures, it can be concluded that all different starting points are optimized to the same value.

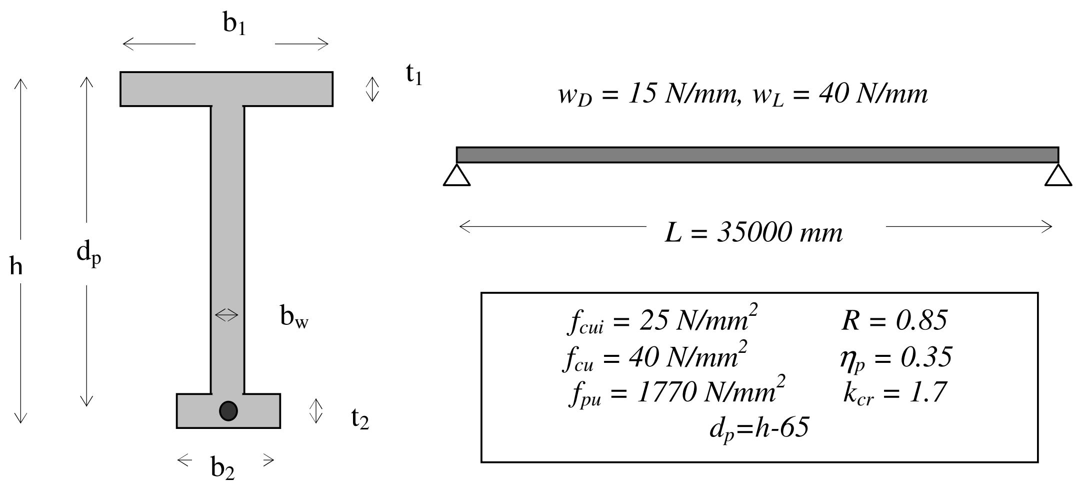


Fig. (11) Simple Beam with I- Cross Section (Example 3)

Table (3) Starting and Optimum Dimensions (Example 3)

Sec.	Starting Point No.		
	I	**II**	**III**
b_w	300	350	400
h	2700	3000	3500
b_1	1700	2000	2200
t_1	200	200	250
b_2	700	900	1100
t_2	200	200	250
Aps	5480.23	5136.45	4547.24
Optimum Values			
b_w	250	250	250
h	4221.88	4221.88	4221.52
b_1	835.39	835.39	835.72
t_1	150	150	150
b_2	250	250	250
t_2	150	150	150
Aps	3256.47	3256.46	3256.64

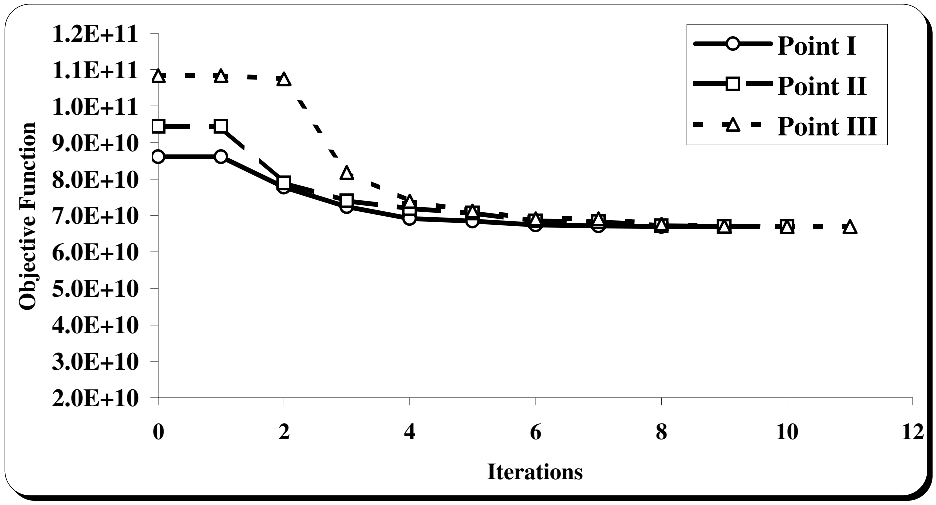


Fig. (12) Development of the Objective Function (Example 3)

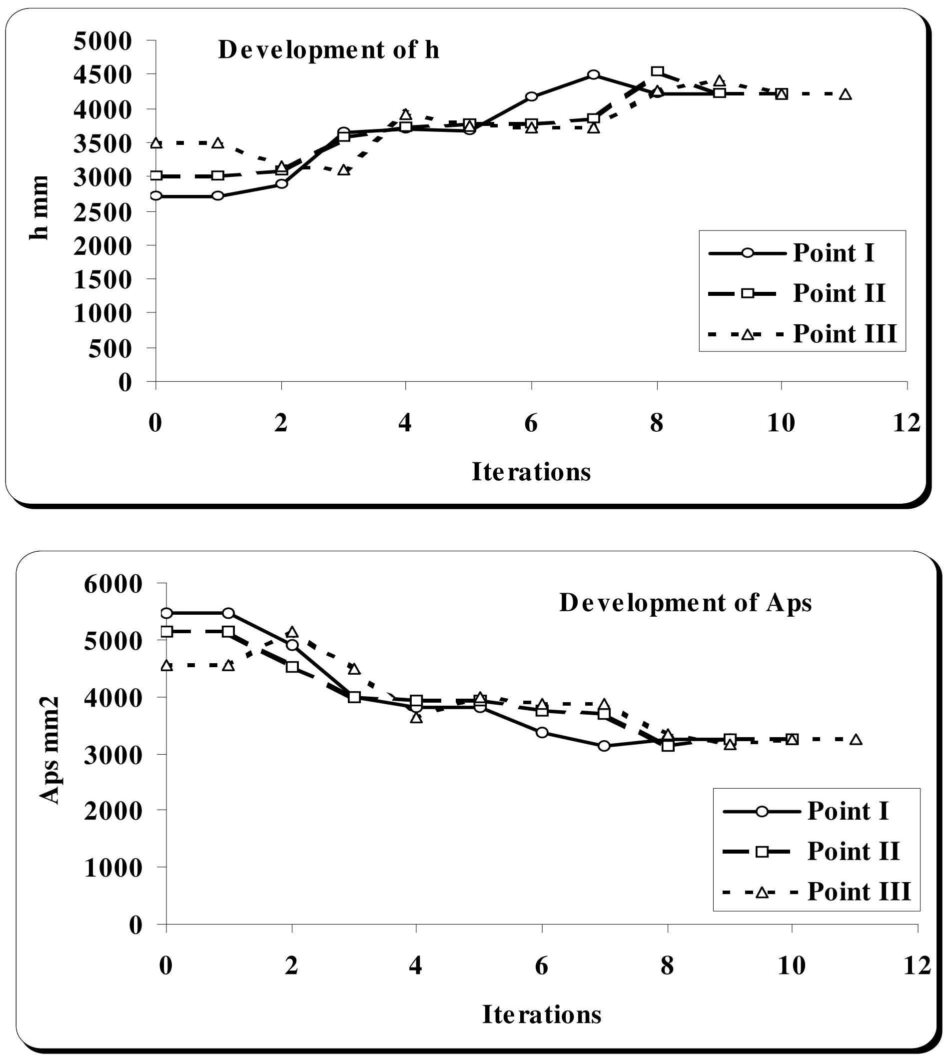


Fig. (13) Development of the Design Variables (Example 3)

Summary and Conclusions

A design procedure incorporating the generalized reduced gradient method is utilized for optimization of prestressed concrete simple beams. The objective function considered is the total materials' cost of the beam. This objective function is minimized subjected to strength and serviceability requirements according to the ECC'01. Three examples with span ranging from 12000 mm to 35000 mm are studied to show the efficiency of the optimization technique. The beams having R-, T-, and I-sections. Three different starting points are applied for each example. The obtained results show that the minimization of the objective function is satisfied perfectly for all starting points.

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NOMENCLATURE

Major symbols are defined below while minor symbols are defined where used:

Symbol	Designation	Units
a	Depth of equivalent rectangular stress block.	mm
A_c	Area of concrete at considered cross section.	mm^2
A_{ps}	Area of prestressed reinforcement in tension Zone.	mm^2
A_s	Area of nonprestressed tension reinforcement.	mm^2
$A_{s`}$	Area of nonprestressed compression reinforcement.	mm^2
A_{st}	Area of shear reinforcement within a distance s.	mm^2
b_1	Width of compression face of member.	mm
c	Distance from extreme compression fiber to the neutral axis.	mm
d	Distance from extreme compression fiber to the centroid of nonprestressed tension reionforcement.	mm
d_p	Distance from extreme compression fiber to the centroid of nonpresested compression reinforcement.	mm
$d`$	Distance from extreme compression fiber to the centroid of non prestressed compression reinforcement.	mm
e	Prestress steel eccentricity.	mm
E_c	Modulus of elasticity of concrete after 28 days.	N/mm^2
E_{ci}	Modulus of elasticity of concrete at time of presterss transfer.	N/mm^2
E_P	Modulus of elasticity of prestressing tendons.	N/mm^2
F_1	The stress at top fiber.	N/mm^2
F_2	The stress at bottom fiber.	N/mm^2
f_{cu}	Characteristic compressive strength of standard concrete cube after 28 days.	N/mm^2
f_{cui}	Characteristic compressive strength of standard concrete cube at time of prestress transfare.	N/mm^2
f_{ctr}	Specified ultimate cracking stress for concrete in tension.	N/mm^2
f_{pci}	Primary stresses in concrete that contacted to the prestressing steel before time dependent losses have been occurred.	N/mm^2
f_{pe}	Stress in prestressed reinforcement after allowance for all prestress losses.	N/mm^2

f_{ps}	Stress in prestressed reinforcement at ultimate strength of the cross section.	N/mm^2
f_{pu}	Specified ultimate tensile strength of prestressing tendons.	N/mm^2
f_{py}	Specified yield strength of prestressing tendons.	N/mm^2
f_y	Specified yield strength of nonprestressed reinforcement.	N/mm^2
h	Overall depth of concrete at the considered cross section.	mm
I_c	Moment of inertia of concrete at considered cross section.	mm^4
k_{cr}	Creep factor for concrete.	
L	The span of the beam.	mm
L'	The length of prestressing tendon.	mm
M_D	The bending moment at midspan due to dead load.	N.mm
M_L	The bending moment at midspan due to live load.	N.mm
M_0	The bending moment due to the self-weight of the member.	N.mm
M_u	The bending moment at midspan due to ultimate load.	N.mm
$\overline{M}_{total}$	The bending moment due to all dead loads plus that due to any portion of the live load that may be considered sustained.	N.mm
M_{total}	The bending moment due to full service load.	N.mm
p_e	Effective prestressing tendon force after allowance for all prestress losses.	N
P_i	Initial prestressing force after allowance for instantaneous losses.	N
p_j	Prestressing jacking force.	N
q_{ci}	Normal shear strength provided by concrete when diagonal cracking results from combined shear and moment.	N/mm^2
q_{cu}	Normal shear strength provided by concrete.	N/mm^2
q_{cw}	Normal shear strength provided by concrete when diagonal cracking results from excessive principal tensile stress in web.	N/mm^2
q_d	Shear stress at section due to unfactored dead load.	N/mm^2
q_p	Shear stress provided by vertical component of effective prestress force at section considered.	N/mm^2
R	The effectiveness ratio (i.e. the percentage of P_i remaining after all time-dependent losses take place).	
s	Spacing of stirrups perpendicular to the axis of the member.	
w_D	Member dead load ,i.e. member self-weight plus superimposed dead load.	N/mm
w_d	Member superimposed dead load.	N/mm
w_0	Member self-weight load.	N/mm
w_L	Member live load.	N/mm
w_u	Member ultimate load.	N/mm
y_t, y_b	The distances between the concrete centroid and the top and bottom fibers of the cross-section, respectively.	mm
γ_c	Strength reduction factor for concrete ($\gamma_c = 1.5$).	
γ_s	Strength reduction factor for nonprestressed reinforcement ($\gamma_s = 1.15$).	
γ_{ps}	Strength reduction factor for prestressing tendons ($\gamma_{ps} = 1.15$).	
Δf_{pe}	Prestressing losses due to elastic shortening.	N/mm^2
Δ_{pi}	Immediate camber due to the initial prestress.	mm
Δ_{wo}	Immediate deflection due to member self-weight.	mm
Δ_d	Immediate deflection due to the superimposed dead load.	mm
Δ_L	Immediate deflection due to the live load.	mm
ρ_c	Concrete density.	N/mm^3

ρ_{ps}	Prestressing tendon density.	N/mm^3
η_p	Factor accounting for the shape of the stress-strain relationship of the prestressing steel. = 0.68, for f_{py}/f_{pu} not less than 0.80 = 0.50, for f_{py}/f_{pu} not less than 0.85 = 0.35 for f_{py}/f_{pu} not less than 0.90	
μ	Ratio of non-prestressed tension reinforcement ($=A_s/b_1d$).	
μ`	Ratio of non-prestressed compression reinforcement ($=A_s'/b_1d$).	
μ”	Shear reinforcement ($=A_{st}/b_w s$)	
μ''_{min}	Minimum shear reinforcement.	
μ_p	Ratio of prestressed reinforcement ($=A_{ps}/b_1d_p$).	
ε_{cu}	Specified ultimate compressive strain of concrete.	
ε_{pu}	Specified ultimate tensile strain of prestressing tendon.	
ε_{py}	Specified yield strain of prestressing tendons.	